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LETTER TO THE EDITOR

Vortex-motion-induced voltage noise in disordered type-II superconducting films

Yoshihisa Enomoto†
Department of Physics, Nagoya Institute of Technology, Gokiso, Nagoya 466, Japan

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Abstract. We explore the voltage noise properties arising from magnetic vortex motion disturbed by both thermal fluctuation and random impurities. On the basis of a Langevin dynamics simulation of the two-dimensional vortex dynamics driven by a transport current in random media, we discuss the relationship between the voltage noise properties and the vortex motion. In particular, it is demonstrated that $1/f$ noise power spectra appear in the plastic flow region of the current–voltage characteristics, where an intermittent behaviour of the vortex dynamics is observed with a large noise amplitude.

It has been recognized that noise measurement for the mixed state of type-II superconductors is a useful tool in the study of the details of vortex dynamics [1]. Indeed, various fascinating properties of the flux flow noise have been observed in recent experiments for driven magnetic vortex systems in conventional and also high-T_c superconductors [2–7], such as $1/f$ noise, an anomalous increase of the noise amplitude, and crossover phenomena exhibited by the noise properties. Several concepts and models have been proposed to explain the experimental data [8]. However, the understanding of the noise behaviour is still incomplete, and there are still many unsolved problems.

In this letter, we discuss the direct link between the noise properties and the collective vortex dynamics, on the basis of a Langevin dynamics simulation of a vortex model [9, 10]. As the first step in the simulations, we here consider the voltage noise due to the two-dimensional vortex motion disturbed by both thermal noise and random impurities. The vortex model is based on the vortex picture, and is described by the stochastic equation of motion for vortex positions [9, 10]. This model has the advantage of allowing one to visualize realistic vortex motion, and thus to obtain direct information on the dynamical properties of vortex systems. In our previous work [11, 12], we have found nonlinear threshold behaviour of the current–voltage (I–V) characteristics for disordered systems: $V \propto (I - I_{th})^a$, with the exponent a and threshold current I_{th} [11]; and Kosterlitz–Thouless-(KT-) type behaviour for two-dimensional clean systems, including the vortex pair creation effect: $V \propto I^b$, with a sudden change of the exponent b at the KT transition temperature [12]. In these simulations, we have only studied the macroscopic properties of the vortex motion—that is, the I–V curves. In order to discuss the voltage noise properties, we here focus on the microscopic (or time-dependent) information on the vortex dynamics.

We consider a thin slab of type-II superconductor, lying on a certain region in the x–y plane, with cross sectional area A. The thickness of the system, w, is taken to be comparable to the magnetic penetration depth, λ. The restriction to the case where $w \simeq \lambda$

† E-mail: eno@ks.kyy.nitech.ac.jp.

might correspond to the cases for thin-film samples and rigid vortices, in which the effects of vortex line bending could be ignored. An external magnetic field is applied along the positive z-direction, and a transport current flows along the positive y-direction. In the present vortex model, the effects of vortex–vortex interaction, random impurities, thermal noise, and the Lorentz force due to the transport current are included. The effects of random impurities are modelled as a random potential [13]. However, the effects of the Magnus force and the additional vortex pair creation have been neglected for simplicity.

The position of the ith vortex at time t is described by the two-dimensional position vector $\boldsymbol{r}_i(t)$. Under these conditions, the free energy for an N-vortex system is given, with conventional notation [14], by

$$F(\{\boldsymbol{r}_l\}) = \sum_l \sum_{i \neq l} V(\boldsymbol{r}_l(t) - \boldsymbol{r}_i(t)) + \sum_{l=1}^{N} V_p(\boldsymbol{r}_l) - \frac{\phi_0 w}{c} j \sum_{l=1}^{N} \hat{\boldsymbol{x}} \cdot \boldsymbol{r}_l \tag{1}$$

where j denotes the transport current density, and $\hat{\boldsymbol{x}}$ is the unit vector along the x-axis with the flux quantum ϕ_0. The intervortex potential, $V(\boldsymbol{r})$, is assumed to be given by

$$V(\boldsymbol{r}) \equiv 2\epsilon(T) K_0\left(\frac{|\boldsymbol{r}|}{\lambda(T)}\right) \tag{2}$$

where

$$\epsilon(T) \equiv (\phi_0/4\pi\lambda(T))^2$$

with $\lambda(T)$ the magnetic penetration depth at temperature T. The random potential, $V_p(\boldsymbol{r})$, represents the effects of quenched random impurities [13], assumed to be characterized by $\overline{V_p(\boldsymbol{r})} = 0$, and

$$\overline{V_p(\boldsymbol{r}) V_p(\boldsymbol{r}')} = \frac{1}{4} n_p a_p \epsilon(T)^2 \left(1 - \frac{T}{T_{c0}}\right)^{-2} \delta(\boldsymbol{r} - \boldsymbol{r}') \tag{3}$$

where the overline denotes the average over the impurity randomness, and n_p is the impurity strength and a_p the area of the unit cell of the underlying crystal lattice, with T_{c0} the superconducting transition temperature at zero field.

Assuming the Langevin dynamics, the equation of motion for the ith vortex position is given by

$$\gamma^{-1} \frac{\mathrm{d}\boldsymbol{r}_i(t)}{\mathrm{d}t} = -\frac{\delta}{\delta \boldsymbol{r}_i} F(\{\boldsymbol{r}_l\}) + \boldsymbol{f}_i(t) \tag{4}$$

where

$$\gamma^{-1} \equiv \sigma H_{c2}(T)\phi_0/c^2$$

is the Bardeen–Stephan friction coefficient given in terms of the normal-state conductivity σ, the upper critical field $H_{c2}(T) \equiv \phi_0/2\pi\xi(T)^2$, and the coherence length $\xi(T)$. The last term in equation (4) describes the thermal noise, assumed to be characterized by $\langle\langle f_{i\alpha}(t)\rangle\rangle = 0$ and

$$\langle\langle f_{i\alpha}(t) f_{l\beta}(t')\rangle\rangle = 2k_B T \gamma \delta_{il} \delta_{\alpha\beta} \delta(t - t')/w \tag{5}$$

where $f_{i\alpha}$ is the αth component of $\boldsymbol{f}_i$, and $\langle\langle \cdots \rangle\rangle$ denotes the average over the thermal fluctuation.

Moreover, we assume the temperature dependence of above two length scales to be given by

$$\lambda(T) \equiv \lambda(0)(1 - T/T_{c0})^{-1/2} \qquad \text{and} \qquad \xi(T) \equiv \xi(0)(1 - T/T_{c0})^{-1/2}$$

at all temperatures. Thus, in this case the Ginzburg–Landau parameter, κ, becomes independent of temperature, and is given by $\kappa \equiv \lambda(0)/\xi(0)$. We also use the relationship obtained from microscopical theory:

$$4\pi\lambda(T)^2\sigma/c^2 = t_0(1 - T/T_{c0})^{-1}$$

with $t_0 \equiv \pi\hbar/(96k_B T_{c0})$ [14]. Note that $t_0 \sim 10^{-14}$ s for $T_{c0} = 10$ K.

Strictly speaking, the present model is defective, at least in the following two features: (i) it is not evident that the condition $w \simeq \lambda$ mentioned above is enough to guarantee straight vortices, at least not for strongly anisotropic superconductors; (ii) the $K_0(x)$ intervortex potential in equation (2) is only valid for very long vortices, i.e., not in the case of a thin film [8]. These points are important in quantitatively comparing simulation results with experimental data, and will be discussed in the future. Although there are these faults, we here adopt the above simple model as a first step in studying vortex-motion-induced voltage noise.

Finally, we comment on the numerical procedure used to solve the stochastic equation (4). First of all, we rewrite equation (4) in terms of dimensionless variables. To do so, we take the unit of length to be $\xi(0)$, the unit of time to be t_0, the unit of magnetic induction to be $H_{c2}(0)$, and the unit of current density to be

$$j_0 \equiv cH_{c2}(0)/(6\sqrt{3}\pi\lambda(0)).$$

Then, we transform the dimensionless equation into a spatial discretized version by using a second-order central-difference representation for the spatial derivatives with dimensionless spacing 1. The final procedure is that of integrating numerically the resulting equation of motion by using a forward Euler difference scheme with dimensionless time step $\Delta\tau$. As boundary conditions, we use periodic boundary conditions in the x–y plane. On the other hand, the initial positions of the vortices are randomly distributed. For more details of numerical methods, as well as a treatment of the thermal noise and the random potential, see our previous work [10–12].

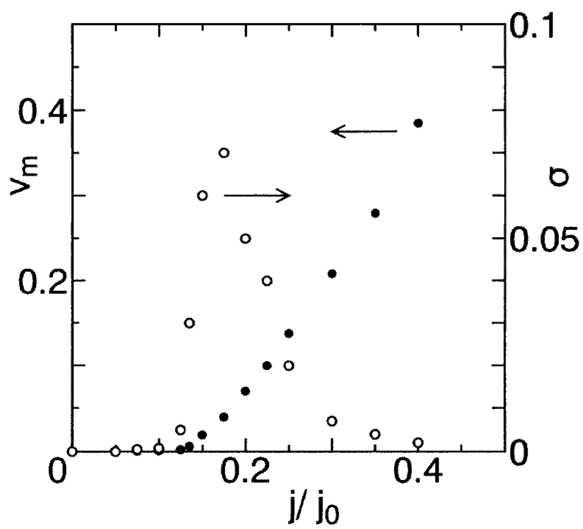

Figure 1. The mean vortex velocity v_m as a function of the transport current density j/j_0, denoted by ●. Also shown is the noise amplitude σ, denoted by ○.

Now we carry out two-dimensional simulations for the vortex motion driven by transport currents in random media. In actual simulations, we choose the parameter values $\kappa = 2$, $n_p = 0.05$, and the energy scale ratio $k_B T_{c0}/(\epsilon(0)w) = 10^{-3}$. We also take $w = \lambda(0)$, $A = 10^2\xi(0) \times 10^2\xi(0)$, and $a_p = \xi(0)^2$. Here we set $N = 500$ and $T/T_{c0} = 0.5$. These values are chosen for the computational reason that they allow us to obtain results efficiently, within our computer availability limitations. The dimensionless time increment $\Delta\tau$ is chosen to be 0.01, which is small enough to ensure numerical stability of the algorithm.

In order to discuss the voltage noise properties, we measure the ensemble-averaged time-dependent vortex velocity $v(t)$ along the x-direction, defined in dimensionless units as

$$v(t) \equiv \left\langle \hat{\boldsymbol{x}} \cdot \frac{\mathrm{d}\boldsymbol{r}_i}{\mathrm{d}t} \right\rangle \frac{t_0}{\xi(0)} \tag{6}$$

where $\langle \cdots \rangle$ denotes the ensemble average over both vortices and ten independent simulation runs. Moreover, to examine the macroscopic motion of the vortex system, we also calculate the mean vortex velocity v_m as

$$v_m \equiv \langle v(t) \rangle_t \tag{7}$$

where $\langle \cdots \rangle_t$ denotes the time average of $v(t)$ over long times up to $10^6 t_0$. In this time averaging, we discard the initial data up to $10^2 t_0$ to avoid the initial transient behaviour. The noise amplitude is described in terms of the standard deviation σ of $v(t)$, defined by

$$\sigma \equiv \sqrt{\langle (v(t) - \langle v(t) \rangle_t)^2 \rangle_t}.$$

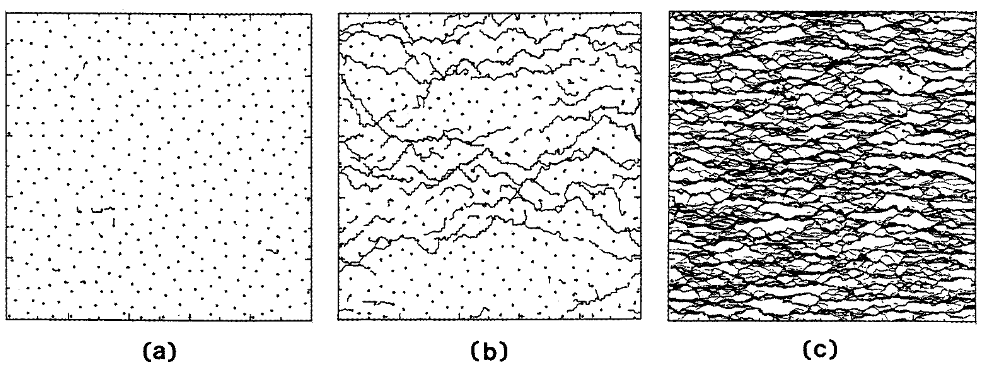

Figure 2. Trajectories of the vortices from $t/t_0 = 1000$ to 1500 for $j/j_0 = 0.1$ (a), 0.175 (b), and 0.4 (c). The dots denote the positions of the vortices at $t/t_0 = 1500$.

In figure 1 we show the mean vortex velocity v_m as a function of the transport current density j. Since the mean velocity v_m obtained is proportional to the electric field along the y-direction induced by the vortex motion, the resulting j versus v_m relation corresponds to the steady-state I–V characteristic. In this figure we also plot the noise amplitude σ as a function of j. As was discussed in previous work [11], the curve is characterized by the nonlinear functional form $v_m \propto (j - j_{th})^a$ near the threshold current density j_{th} ($j_{th} \simeq 0.122 j_0$ and $a \simeq 2.1$ in this case). Moreover, the noise amplitude σ is found to have a peak slightly above the threshold current j_{th}. Next, we examine the crossover phenomena of the vortex flow pattern according to the transport current. In figure 2 we show the vortex trajectories for three typical values of the transport current: $j/j_0 = 0.1$ (a), 0.175 (b), and 0.4 (c). As has been discussed in references [9, 11], we can see the different types of

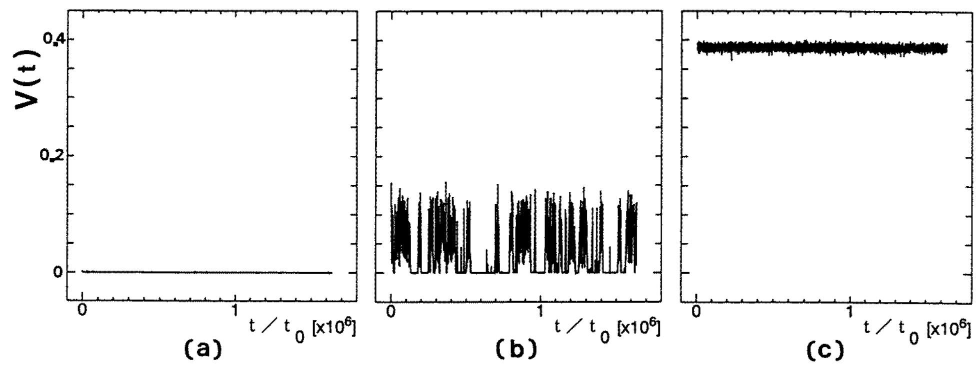

Figure 3. The ensemble-averaged vortex velocity $v(t)$ as a function of time t/t_0 for $j/j_0 =$ 0.1 (a), 0.175 (b), and 0.4 (c).

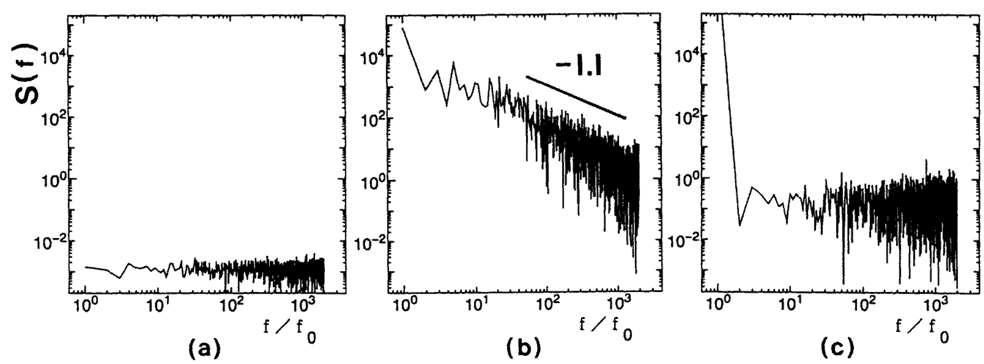

Figure 4. The power spectrum $S(f)$ of the ensemble-averaged vortex velocity $v(t)$ as a function of the frequency f/f_0 for $j/j_0 = 0.1$ (a), 0.175 (b), and 0.4 (c). In (b), a straight line is also plotted with its slope indicated.

vortex motion that occur according to the value of j. In particular, case (b) is called the plastic flow state. In the present simulations, we have observed the plastic flow pattern for $0.135 \leqslant j/j_0 \leqslant 0.225$, where the noise amplitude is relatively large as is shown in figure 1. The ensemble-averaged vortex velocity $v(t)$ is shown in figure 3 as a function of time t for $j/j_0 = 0.1$ (a), 0.175 (b), and 0.4 (c). It is found that for case (b) (in the plastic flow region) intermittent behaviour of the collective vortex motion appears, and this leads to the nonlinear relationship between j and v_m with the large noise amplitude. Finally, to facilitate discussion of the voltage noise properties, we examine the power spectrum, $S(f)$, of the ensemble-averaged time-dependent vortex velocity $v(t)$ as a function of a frequency f, defined by

$$S(f) \equiv \left| \int v(t) \mathrm{e}^{-\mathrm{i}2\pi f t} \, \mathrm{d}t \right|^2 . \tag{8}$$

Since $v(t)$ is proportional to the time-dependent induced voltage, the resulting $S(f)$ is proportional to the voltage noise power spectrum. The power spectra $S(f)$ are shown in figure 4 for $j/j_0 = 0.1$ (a), 0.175 (b), and 0.4 (c). Here, the unit of the frequency is taken as $f_0 = 1/(2^{12} \times 200 t_0)$, whose typical value is estimated to be 10^7 Hz for $T_{c0} = 10$ K. In these figures, we can see the $1/f$ noise power spectrum (estimated as $S(f) \sim f^{-1.1}$)

for $j/j_0 = 0.175$, while the white-noise properties are observed for the small ($j/j_0 = 0.1$) and the large current ($j/j_0 = 0.4$). In the present simulations, $(1/f)$-type behaviours of $S(f)$ (estimated as $S(f) \sim f^{-\nu}$ with $\nu = 0.9$–1.2) have been obtained in the plastic flow region between the white-noise regions for both small and large transport current densities. Similar crossover behaviour of the voltage noise power spectra as a function of the transport current has been obtained experimentally for low-T_c superconducting thin films [2]. However, in this work we cannot find the Lorentzian-type behaviour of the noise power spectra like $S(f) \simeq S_0/(f^2 + f_1^2)$ with positive constants S_0 and f_1 which has been observed experimentally for the intermediate-current region between the $1/f$ noise region and the white-noise region for the large current [2]. In order to examine the reason for this discrepancy, further simulations are now under way that involve carefully changing the values of the temperature, the vortex number, and the pinning strength.

In conclusion, after performing two-dimensional Langevin dynamics simulations of the vortex model driven by the transport current in random media, we have discussed the vortex-motion-induced voltage noise properties as functions of the transport current. The results have demonstrated that the voltage noise properties are in close correlation with the dynamical behaviour of vortex systems. In particular, the $1/f$ noise has been found to appear in the plastic flow region in which intermittent vortex motion occurs with a large noise amplitude.

Here we have restricted the present simulations to the study of the voltage noise arising from the two-dimensional vortex motion disturbed by both thermal noise and random impurities. Recent experiments have revealed that the flux flow noise is generated from many sources, and is affected by various features [3–7], such as the three dimensionality of the vortex lines (causing, e.g., bending, entanglement, and cutting), grain boundary effects, various types of pinning centre, the KT-type vortex pair creation, and the dynamical transitions of the vortex line lattices. Although the results obtained here cannot be directly applied to these cases, the present study might give a useful guide for use when discussing the noise properties in such complicated situations. Moreover, the $1/f$ noise can be widely seen for a number of physical systems [15], and is now discussed in terms of the concept of self-organized criticality [16, 17]. In order to study the generic properties of the noise, it would be interesting to compare the present model simulations with those for other physical systems.

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